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AN ANALYSIS OF THE EFFECTS OF VARIOUS PARAMETERS ON THE AVERAGE VOID FRACTIONS IN SUBCOOLED BOILING

P. G. KROEGER[†] **and N. ZUBER**[‡]
Research and Development Center, General Electric Company, Schenectady, N.Y., U.S.A.

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Abstract—An evaluation of the effects of various parameters on the ability to predict accurately the volumetric vapor concentration in subcooled boiling is presented. It is shown that: (1) the distribution parameter is of major importance as thermodynamic equilibrium is approached; (2) the point of incipient vapor formation greatly affects the void fraction in the subcooled regime—no reliable method for predicting this point exists at this time, and (3) it appears that the axial temperature distribution is of lesser effect.

It is concluded, therefore, that the ability to predict the point of incipient vapor formation is mandatory for accurate prediction of the void fraction in subcooled boiling. Future research should thus be directed towards defining and predicting the location of this point.

The results of the analysis also show that the conditions of the heated surface and the introduction of additives may have two effects on the void fraction in subcooled boiling. These effects are discussed and a method for taking them into account is presented.

Satisfactory agreement is shown between predicted void fractions and recent experimental data obtained in subcooled boiling of Refrigerant-22 and of water over a wide pressure range.

NOMENCLATURE

MLTθ system of units with H = ML^2/T^2

A_c,	cross-sectional area [L^2];
C_0,	distribution parameter;
c_p,	specific heat [H/Mθ];
G,	mass flow [M/L^{2}T];
g,	acceleration due to gravity [L/T^2];
i,	enthalpy [H/M];
Δi_{fg},	latent heat of vaporization [H/M];
j,	volumetric flux of the mixture [L^3/TL^2];
j_f,	volumetric flux of the fluid [L^3/TL^2];
j_g,	volumetric flux of the gas [L^3/TL^2];
L,	length of heated section [L];
$\Delta \ell$,	axial length along heated section from z_0 to z_1 [L];
m,	exponent for radial flux profile equation;
n,	exponent for radial void profile equation;
p_r,	reduced pressure;
q,	heat flux [H/TL^2];
r,	radial distance in a tube [L];
R,	tube radius [L];
T,	temperature [θ];
T_i,	liquid bulk temperature at the inlet of the heated duct [θ];
T_L,	liquid bulk temperature [θ];
T_0,	liquid bulk temperature at z_0 [θ];
T^+,	dimensionless liquid bulk temperature, $T_L - T_0/\Delta T_s$;
ΔT_s,	temperature difference ($T_{sat} - T_0$) [θ];
$\Delta T_{s\,in}$	inlet subcooling ($T_{sat} - T_i$) [θ];
v,	velocity [L/T];
v_{fi},	liquid inlet velocity [L/T];
V_{gj},	drift velocity of the vapor [L/T];
$\bar{V}_{gj}$,	weighted mean vapor drift velocity [L/T];

[†] Present address: Casting Laboratory, Chase Brass and Coper Co., Cleveland, Ohio.

[‡] Present address: Dept. Mechanical Engineering, New York University, University Heights, New York, N.Y. 10453.

x,	vapor quality;
x_{eq},	vapor quality assuming thermodynamic equilibrium;
z_1,	axial coordinate at point of $T_L = T_{sat}$, assuming thermodynamic equilibrium;
z,	axial coordinate at point along heated section measured from the inlet [L];
z_0,	axial coordinate at point of incipient vapor formation [L];
z^+,	dimensionless axial coordinate $z - z_0/\Delta\ell$;
z^*,	dimensionless axial coordinate z/L;
α,	vapor volumetric concentration;
Γ,	rate of mass formation [M/TL^3];
ξ_h,	heated perimeter [L];
ρ,	mass density [M/L^3];
$\Delta\rho$,	density difference $(\rho_f - \rho_g)$ [M/L^3];
σ,	surface tension [ML/T^{2}L];
$\langle\ \rangle$,	value averaged over the cross-sectional area.

Subscripts

c,	at the center line;
CH,	value along a chord of a tube;
DIA,	value along the diameter of a tube;
f,	liquid;
g,	vapor;
i,	at the inlet of the heated section;
r,	relative;
sat,	at saturation;
w,	at the wall.

1. INTRODUCTION

THE ABILITY to predict accurately the vapor void fraction in subcooled boiling is of considerable interest to nuclear reactor technology. Both the steady-state performance and the dynamic response of a reactor depend on the void fraction. In particular, several studies of the dynamic behavior of a two-phase mixture have revealed that the stability of the system depends to a great extent upon the power density and the void behavior in the subcooled boiling region.

In view of the foregoing, an analysis was conducted with the purpose of deriving an equation that can be used to predict the vapor void fraction in a two-phase mixture in thermodynamic non-equilibrium. The results of that study have been reported in [1, 2] which lists also, the more significant investigations that preceeded it.

It was shown in [1, 2] that four effects must be taken into account in an analysis concerned with predicting the vapor void fraction in subcooled boiling. These are the effects: (1) the local relative velocity; (2) the concentration profile; (3) the flow profile, and (4) the temperature profile. In [1, 2] these four effects have been accounted for by means of the vapor drift velocity $\overline{V}_{gj}$, the distribution parameter C_0 and the dimensionless axial temperature distribution T^+.

The equation derived in [1, 2] for predicting the average void fraction (averaged over the cross-sectional area of the duct) was given by:

$$\langle\alpha\rangle = \frac{\dfrac{q}{\rho_g \Delta i_{fg}}\dfrac{\xi_h(z - z_0)}{A_c} - \dfrac{v_{fi}\rho_f c_p \Delta T_s}{\rho_g \Delta i_{fg}} T^+}{C_0 \dfrac{\Delta\rho}{\rho_f}\left\{\dfrac{q}{\rho_g \Delta i_{fg}}\dfrac{\xi_h(z - z_0)}{A_c} - \dfrac{v_{fi}\rho_f c_p \Delta T_s}{\rho_g \Delta i_{fg}} T^+\right\} + \left\{C_0 v_{fi} + \overline{V}_{gj}\right\}\left\{1 + \dfrac{c_p \Delta T_s}{\Delta i_{fg}}(1 - T^+)\right\}}. \tag{1}$$

where the symbols are defined in the nomenclature.

In equation (1), the coordinate z_0 defines the location in the duct where voids are first found on the heated wall, whereas $\Delta T_s = T_{sat} - T_0$, is the bulk subcooling at that point. Equation (1) indicates therefore, that in order to predict the vapor void fraction in subcooled boiling, it is necessary to determine first, the conditions for incipient vapor generation. This, indeed is not surprising.

The results predicted by equation (1) have been compared in [1, 2] to the experimental data available at that time. Satisfactory agreement was reported. However, the importance of the various parameters which appear in equation (1) have not been investigated in detail. This last aspect is important for planning of future experimental and analytical investigations. Furthermore it is important also, when comparing the predicted results with experimental data.

2. PURPOSE OF THE PAPER

This paper has two purposes. The first one is to evaluate the effects of the various parameters on the ability to predict accurately the average value of the vapor void fraction in the subcooled boiling as well as in boiling liquids at saturation temperature. The second objective is to compare the predicted results with experimental data which have appeared in the literature following the writing of [1, 2].

The parameters whose effects are evaluated in this paper are: the distribution parameter C_0, the axial temperature distribution T^+, and the location of the point of incipient vapor generation z_0. As noted above, these parameters take into account the effects of the flow and concentration profiles and of the thermodynamic non-equilibrium of the mixture.

It will be seen that the results of this evaluation define the range of operating conditions in which the effect of a particular parameter is most pronounced in determining the average value of the void fraction $\langle\alpha\rangle$. Information of this kind is desirable for predicting the void fraction, for analyzing available experimental data and for planning of future experiments.

3. PHYSICAL VARIABLES AFFECTING THE VAPOR VOLUMETRIC CONCENTRATION

It was noted in [1, 2] that a flowing two-phase mixture in thermodynamic non-equilibrium is a medium where the temperature, the volumetric concentration and the velocity profiles are varying. If one recalls the characteristics of the single-phase multi-component flow systems with heat and mass transfer, we find that they are determined by specifying the temperature, concentration, and the velocity profiles. It should not be surprising therefore, to see that the characteristics of a two-phase flow system will also depend upon these three parameters. However, since in two-phase mixtures the velocities of the two phases are not equal, a fourth parameter, which accounts for the relative velocity, must be taken into consideration when analyzing a two-phase flow problem. In what follows, we shall first discuss the significance of these four variables and the way they have been accounted for in the analysis of [1, 2]. This will then be followed by a quantitative evaluation of their effect on the void fraction.

3.1 *Local relative velocity*

The local relative velocity between the two phases depends upon the drag exerted at the vapor–liquid interface as well as upon the geometry of this interface. It can be expected, therefore, that the relative velocity will vary whenever the topology of the two-phase mixture changes. Such changes from dispersed spherical to separated cylindrical and back to dispersed spherical interface geometry occur when the flow regime of the two-phase mixture changes from bubbly to annular and from annular to mist (fog) flow regime respectively.

In [1–5], the effect of the relative velocity was taken into account by means of the vapor drift velocity defined as the vapor velocity relative to the velocity of the center of volume of the mixture. The expressions for the drift velocities appropriate to the various flow regimes are given in [1–5]. For the bubbly churn flow regime, of interest to this paper, the weighted mean vapor drift velocity is given by:

$$\overline{V}_{gj} = 1{\cdot}41\left[\frac{\sigma g \Delta\rho}{\rho_f^2}\right]^{\frac{1}{4}}. \tag{2}$$

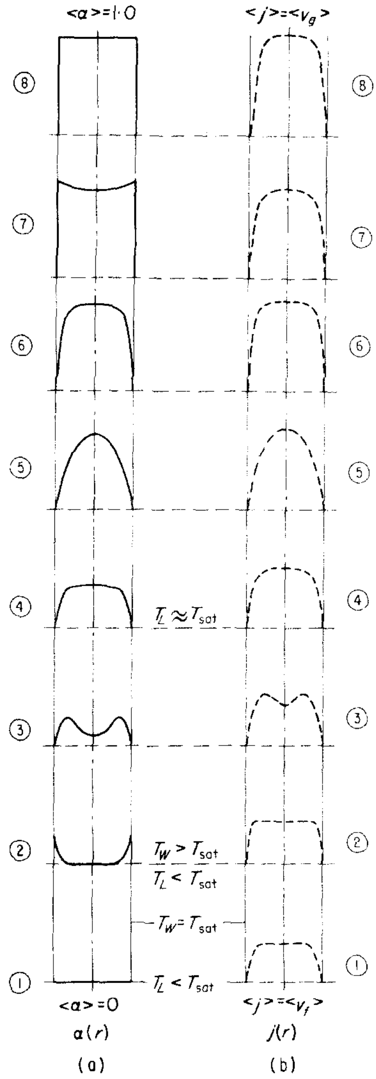

FIG. 1. Variation of concentration and flow profiles along a uniformly heated duct. (a) Volumetric concentration of the vapor; (b) volumetric flux of the mixture.

3.2 *Concentration and flow profiles*

In order to understand the effect of radial void and flow profiles on the average void fraction $\langle\alpha\rangle$ it is instructive to examine the variation of these profiles along the axial coordinate in a uniformly heated duct. Such a variation is shown schematically in Fig. 1(a) for the void profile $\alpha(r)$, and in Fig. 1(b) for the volumetric flux profile of the mixture $j(r)$ (defined as the sum of the volumetric fluxes of the liquid and of the vapor).

Figure 2 is an example of experimental data obtained in this investigation, which show the concentration profiles as function of quality and of subcooling. Since these experiments have been performed with an axially uniform power input, the functional dependence of $\langle\alpha\rangle$ upon quality and subcooling can be expressed also as function of distance. Similar profiles obtained with other fluids are shown in [5]. We note that simultaneously recorded data on concentration and flow profiles in subcooled boiling have not been reported in the literature. Consequently, some of the $j(r)$ profiles shown on Fig. 1(b), still need experimental verification.

Consider now the profiles shown in Fig. 1. For a degassed fluid with a sufficiently high inlet subcooling, no bubbles will be present at the entrance of the heated section (station 1). Consequently, at this station $\langle\alpha\rangle$ will be zero whereas the volumetric flux profile of the mixture will correspond to the velocity profile of the liquid only. For turbulent flow, the latter will have the shape shown on Fig. 1(b).

The temperature of the wall T_w, will reach the value of the saturation temperature somewhere downstream from the inlet. From this point on bubbles can be generated on the heated wall. The exact location where this process starts will depend on the nucleating characteristics of the surface as well as on the surrounding flow and temperature fields.

Following the nucleation, bubbles grow on the heated surface. However, since the bulk liquid temperature is still below saturation, bubbles will either collapse on the surface or detach and collapse in the bulk liquid. Consequently, the vapor concentration profile will decrease from a given value at the wall to zero at the centerline (see station 2). Whether bubbles slide along the wall or detach and move parallel to the wall as they collapse in the bulk, they will contribute to the axial volumetric flux of the mixture. Since this contribution will be most

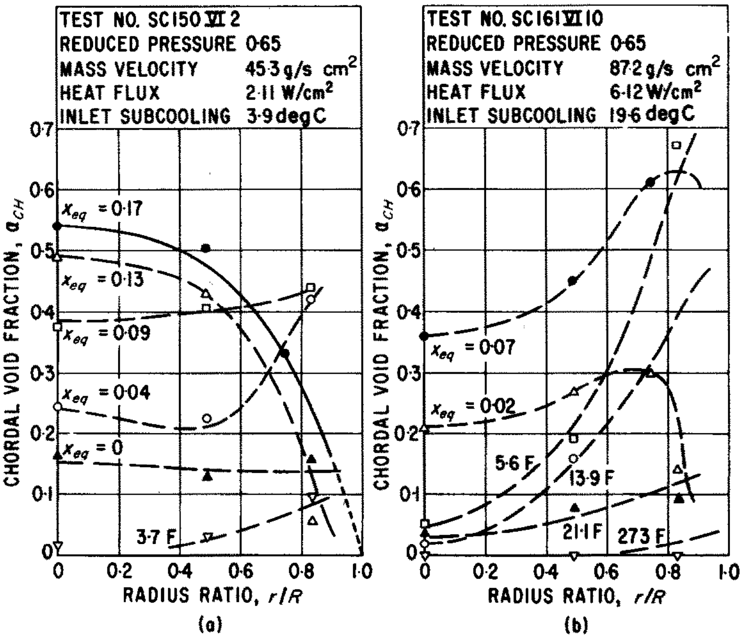

FIG. 2. Experimental chordal void fractions in subcooled boiling.

noticeable in the vicinity of the heated wall, the $j(r)$ profile will be flatter at station 2 than at station 1.

In the bulk of the liquid, the rate of bubble collapse decreases as the temperature increases. Because of the radial temperature distribution in the liquid and because of the bubble migration toward the center, the concentration profile will exhibit two maxima and a minimum point (see station 3). At pressure levels where the density of the vapor is considerably lower than the density of the liquid, the volumetric flux of the mixture will be greatly affected by the volumetric flux of the vapor. Consequently, at these pressures, it can be expected that the $j(r)$ profile will be similar to that of $\alpha(r)$ as indicated at station 3 on Fig. 1(b).

When the bulk liquid reaches saturation temperature, the bubble collapse will cease, whereas the migration toward the center will continue. Consequently, the concentration profile will become first flat as shown at station 4 and then more pronounced as the bubbly flow regime fully develops further downstream (see station 5).

As evaporation along the duct continues, and the vapor void fraction together with the volumetric flux increases, the flow regime will change from bubbly churn turbulent to the annular flow regime. As a result of this change of interface geometry and re-distribution of phases, the concentration as well as the flux profiles will flatten out (see station 6).

Further downstream, as droplets become entrained in the vapor core, the flow regime will change to annular-mist flow. For sufficiently long ducts, the liquid film adjacent to the heating wall may disappear as a consequence of evaporation and/or of drop entrainment. When this occurs, the flow regime changes to mist flow in which the only liquid present is in the form of droplets concentrated in the central region of the vapor core. For such a flow regime, the vapor concentration is highest at the heated wall; indeed it may have there, the value of unity for a dry wall. The lowest vapor concentration is in the central region where droplets are present. Such a vapor concentration profile is shown at station 7 together with the corresponding flux profile of the mixture. The latter does not change the curvature because of the requirement that the vapor velocity must be zero at the wall.

Finally, for complete vaporization as encountered in once-through systems, the vapor concentration becomes unity, whereas the profile of the volumetric flux of the mixtures becomes equal to the velocity distribution of the vapor phase only, as shown at station 8.

It can be seen from the preceding discussion that in a two-phase system with heat addition, the concentration and the flux profiles change along the duct as evaporation takes place. Since it is well known from studies of single phase flow systems with mass transfer that the shapes of the concentration and of the velocity profiles have an important effect on the value of the average concentration, it can be expected that similar results will be evident in vapor–liquid mixtures. In particular, it can be expected that when the vapor is concentrated in the low velocity region (station 2 and 7), it will be transported at a velocity that is lower than the average velocity $\langle j \rangle$. Conversely, when voids are concentrated in the high velocity region (station 5), they will move faster than $\langle j \rangle$. This is indeed the case as shown in [3].

It was shown in [1–6] that the concentration and velocity profiles affect the average void fraction through a *distribution parameter* C_0, defined by

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \tag{3}$$

where α and j are the radial profiles of concentration and of flow respectively.† By examining equation (1), it can be seen that, with all other parameters being equal, an increase of C_0 results in a decrease of the average void fraction $\langle \alpha \rangle$ and vice versa.

The concentration and flux profiles analyzed in [1–5] were of the form

$$\frac{\alpha - \alpha_w}{\alpha_c - \alpha_w} = 1 - \left(\frac{r}{R}\right)^n \tag{4}$$

$$\frac{j}{j_c} = 1 - \left(\frac{r}{R}\right)^m \tag{5}$$

where the subscripts c and w denote quantities evaluated at the center line and at the wall respectively. The reason for selecting these profiles was their simplicity as well as their accuracy in representing the experimental data [2, 5]. It can be seen that equation (4) and equation (5) describe the profiles shown at stations 2, 4 through 8. Clearly, a more complex expression is required to describe the profiles at station 3.

With the profiles given by equation (4) and equation (5), the distribution parameter C_0 becomes

$$C_0 = 1 + \frac{2}{m + n + 2}\left[1 - \frac{\alpha_w}{\langle \alpha \rangle}\right] \tag{6}$$

when expressed in terms of α_w and

$$C_0 = \frac{m + 2}{m + n + 2}\left[1 + \frac{\alpha_c}{\langle \alpha \rangle}\frac{m}{m + 2}\right] \tag{7}$$

when expressed in terms of α_c.

It can be seen from the preceding discussion for a radially *uniform* concentration ($\alpha_c = \alpha_w = \langle \alpha \rangle$) the distribution parameter is unity, i.e. $C_0 = 1$.

Equation (7) shows that when α_c is zero, i.e. when the voids are concentrated in the low velocity region (see station 3) then $C_0 < 1$. Assuming all other parameters being equal, equation (1) indicates that a system with such profiles will have a higher value of $\langle \alpha \rangle$ than a system with a radially uniform concentration. Conversely, lower values of $\langle \alpha \rangle$ are to be expected in a system where voids are concentrated in the high velocity region (station 5) because for

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† The inverse of C_0 was first introduced in [6] where it was referred to as the "flow parameter". In that reference, the effects of neither the local relative velocity nor of the subcooling were considered.

such profiles, equation (6) shows that $C_0 > 1$. It was shown in [3, 4] that, for profiles given by equations (4) and (5), the maximum value of C_0 is 1·5, which corresponds to peaked profiles at the center. For flat profiles (see stations 4 and 6) the value of C_0 tends to unity. The changes of the concentration profiles shown in Fig. 2 as well as those shown in [5] indicate that such variations of C_0, and the attendant effects on $\langle\alpha\rangle$, can be expected as vaporization takes place and flow regimes change along the heated duct.

3.3 *Thermodynamic non-equilibrium*

In order to determine the vapor volumetric concentration, it is necessary to determine first, the volumetric flux of the vapor. In subcooled boiling, the latter depends on the rate of bubble nucleation and on their subsequent growth and collapse. Were the equations describing these processes known and/or sufficiently reliable, they could be used to formulate the appropriate *constitutive equation of* evaporation which, together with the energy equation of the mixture, would specify the vapor flux [1, 2].†

However, since neither the process of nucleation nor the process of bubble growth and collapse can be predicted reliably, it is desirable and advantageous to determine the vapor volumetric flux from the energy equation of the mixture by assuming an appropriate axial temperature distribution [1, 2]. This latter assumption is equivalent to specifying the appropriate form for the state of thermodynamic non-equilibrium of the mixture for a given operating condition. Such an assumption is required because in subcooled boiling, part of the energy which is transferred from the heated wall is used to increase the temperature of the subcooled liquid whereas the other is used for vaporization. This is illustrated in Fig. 3 which shows the quality and the liquid bulk temperature as function of distance for a system with an axially uniform power input.

Let us consider first, a system in thermodynamic equilibrium. By definition, in such a system neither can vapor be present in the subcooled liquid region nor can the liquid become superheated. Consequently, in the subcooled region the entire energy transferred to the system is used to increase the enthalpy of the liquid to saturation; beyond that point it is used entirely for vaporization. In such a system, therefore, the bulk liquid temperature will increase linearly with distance until it reaches saturation at a point $z = z_1$, thereafter remaining constant. Furthermore, up to point z_1, there will be no vapor generation; thereafter the vapor flux and therefore, the quality will increase linearly with distance (see Fig. 3).

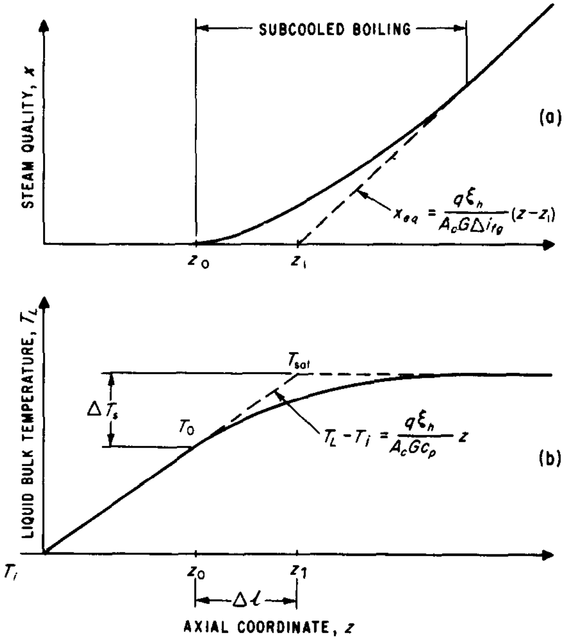

FIG. 3. Schematic presentation of liquid temperature and vapor quality in a uniformly heated duct.

A real system, however, will not be in thermodynamic equilibrium because vapor generation may start already at point z_0, where the temperature of the wall exceeds saturation although the bulk liquid temperature T_0 is below T_{sat}, i.e. the

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† This approach is discussed further in Section 6 of this paper.

bulk liquid is still subcooled. From this point on, the vapor flux and therefore, the quality increase with distance although, not linearly. In this region, part of the energy transferred from the heating wall is used to increase the enthalpy of the liquid, the other is used for vaporization. Consequently, past point z_0, the increase of the bulk liquid temperature will cease to be linear with distance. Since the rate of temperature rise is decreased, the bulk will reach saturation temperature further downstream from point z_1. Beyond that point, the vapor flux and the quality will increase again linearly with distance.

It is evident from the foregoing that the axial quality and temperature distributions are related. It is evident also, that either distribution can be used to describe the state of thermodynamic non-equilibrium. In [1, 2], this state was described by assuming the temperature distribution. The boundary conditions and the form which this distribution must satisfy are given in [1, 2]. Among the various functions which would satisfy these conditions two were selected and were used in equation (1) to predict $\langle\alpha\rangle$. These functions, chosen in view of their simplicity, were:

$$T^+ = \frac{T_L - T_0}{T_{\text{sat}} - T_0} = 1 - \exp\left[-\frac{z - z_0}{\Delta\ell}\right] \tag{8}$$

and

$$T^+ = \frac{T_L - T_0}{T_{\text{sat}} - T_0} = \tan h\left[\frac{z - z_0}{\Delta\ell}\right] \tag{9}$$

where the characteristic length $\Delta\ell$ is given by

$$\Delta\ell = z_1 - z_0 = \frac{A_c}{q\xi_h} v_{fi}\rho_f c_p\,[T_{\text{sat}} - T_0]. \tag{10}$$

The relation derived in [1, 2] between the axial quality distribution and the temperature distribution was given by:

$$G_g = xG = \rho_g\langle j_g\rangle$$

$$= \rho_g\left\{\frac{\dfrac{q}{\rho_g\Delta i_{fg}}\,\dfrac{\xi_h(z - z_0)}{A_c} - \dfrac{v_{fi}\rho_f c_p\Delta T_s}{\rho_g\Delta i_{fg}}T^+}{1 + \dfrac{c_p\Delta T_s}{\Delta i_{fg}}(1 - T^+)}\right\}. \tag{11}$$

It can be seen from equations (8–11), as well as from equation (1), that in order to compute the quality and the average void fraction, it is necessary to determine first the location of the point z_0 where vapor generation starts. Before discussing this aspect of the problem, it will be instructive to examine the effects of thermodynamic non-equilibrium on the void profiles as well as on the average void fraction.

Figures 4 and 5 show measured average void fractions and those computed from equation (1) as a function of axial distance for the test conditions of Fig. 2. Above each experimental value of $\langle\alpha\rangle$ on these two figures, there is a sketch of the corresponding profile in Fig. 2, together with the value of the bulk liquid temperature T^+ at that point computed from equation (8). The location of z_0 shown in these figures was estimated from the experimental data, whereas z_1 was computed. Note that this latter point corresponds to the location where the quality based on thermodynamic equilibrium is zero.

Two observations can be made with respect to Figs. 4 and 5. First, it can be seen that the concave and double profiles which are characteristic of subcooled boiling persist well beyond the point z_1 where $x_{eq} = 0$. This could be interpreted as an indication that, because of thermodynamic non-equilibrium, the bulk liquid temperature reaches saturation downstream from the point z_1. It is noteworthy that the change to convex profiles, which are characteristic of the fully developed bubbly regime, takes place where the computed bulk liquid temperature is close to saturation, i.e. $T^+ \sim 0{\cdot}96$. Second, Fig. 5 shows that there is almost no change in the value of $\langle\alpha\rangle$ between the two locations, where $T^+ = 0{\cdot}92$ and $T^+ = 0{\cdot}96$. This result could be attributed to some experimental error were it not for the fact that similar behavior was also observed in several other experimental runs. Such a behavior of $\langle\alpha\rangle$ could be accounted also by the change of profiles from uniform at $T^+ = 0{\cdot}92$ to convex at $T^+ = 0{\cdot}96$. It was discussed above that, with all other parameters being equal, radially uniform profiles will give higher values of $\langle\alpha\rangle$

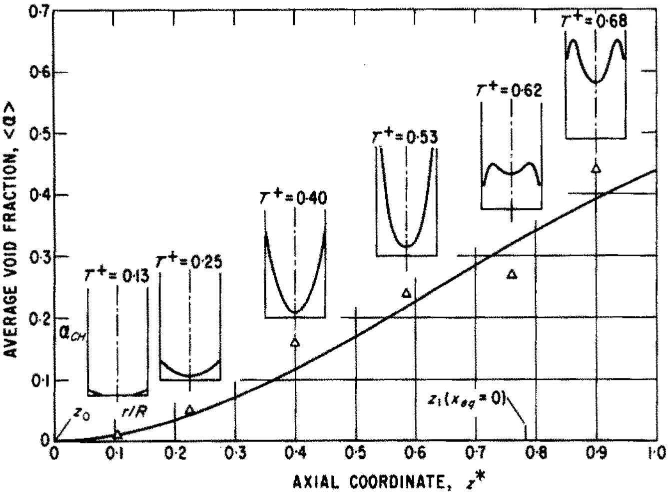

FIG. 4. Axial void distribution in subcooled boiling with corresponding void profiles at various stations (for details on void profiles and test conditions see Fig. 2b).

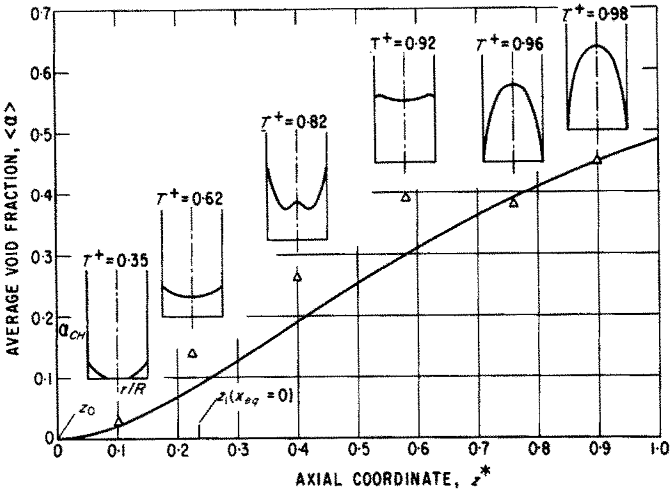

FIG. 5. Axial void distribution in subcooled boiling with corresponding void profiles at various stations (for details on void profiles and test conditions see Fig. 2a).

than convex profiles. Although this explanation is still tentative, the results shown on Figs. 4 and 5, as well as those reported in [5] clearly indicate the interaction between the concentration and temperature profiles and their effect on the average void fraction.

3.4 *Point of initial vapor formation*

It was noted above that for degassed liquids, the vapor generation can start as soon as the temperature of the wall exceeds saturation, i.e. as soon as the liquid adjacent to the wall becomes superheated. The amount of superheat required to generate the vapor depends on the nucleating characteristics of the surface as well as on the flow and temperature field. For degassed therefore, the location of z_0 will be downstream from the point $z_{w\,\mathrm{sat}}$ where $T_w = T_{\mathrm{sat}}$.

Furthermore, for liquids that cannot maintain a large bulk liquid superheat† z_0 will not exceed z_1. The problem is then reduced to locate accurately z_0 between these two limits, i.e. between $z_{w\,\mathrm{sat}} \leqslant z_0 \leqslant z_1$.

Several criteria for predicting vapor formation have been reported, [7–9] among others; some of these predict the conditions required to nucleate the first bubble, others predict the conditions for bubbles to detach from the wall and rise in the subcooled main stream. Whether any of these conditions can be related to z_0, determined from void measurements remains to be proven.

In [1, 2] some of the available criteria were used to compute z_0. With water at elevated pressures flowing through rectangular channels, the values predicted by the criterion of [9] were sufficiently close to the points where the voids were first detected experimentally. However, the agreement with experimental data was not very satisfactory at low pressures and for fluids other than water. It was concluded in [1, 2] that a criterion which could be used to predict z_0 for various fluids and operating conditions is not yet available.

4. EFFECTS OF VARIOUS PARAMETERS

The influence which various physical variables have on the average void fraction were discussed in the preceding section. In what follows we shall evaluate quantitatively their effect.

In order to analyze the various effects it is advantageous to rewrite equation (1) in the form of:

$$\langle\alpha\rangle = \frac{x}{C_0 \dfrac{\Delta\rho}{\rho^f} x + \left[C_0 + \dfrac{\overline{V}_{gj}}{v_{fi}}\right] \dfrac{\rho_g}{\rho_f}}, \tag{12}$$

where the quality is given by equation (11). This latter can be also expressed as:

$$x = \frac{q\xi_h \Delta\ell}{G A_c \Delta i_{fg}} \left\{ \frac{z^+ - T^+}{1 + \dfrac{c_p \Delta T_s}{\Delta i_{fg}} (1 - T^+)} \right\}, \tag{13}$$

or as

$$x = \frac{c_p \Delta T_s}{\Delta i_{fg} + c_p \Delta T_s (1 - T^+)} (z^+ - T^+), \tag{14}$$

where

$$z^+ = \frac{z - z_0}{\Delta\ell}. \tag{15}$$

Using these expressions the effects of the distribution parameter C_0, of the axial temperature distribution T^+, and of the initial point z_0, can be evaluated by varying these parameters in equation (12). The results of such an evaluation are discussed below.

4.1 *Distribution parameter*

Figure 6 shows the computed values of $\langle\alpha\rangle$ for given z_0 and T^+ as function of the distribution parameter C_0. The range of variation of C_0 used in preparing this figure covers the profiles discussed in Section 3.2 and illustrated in Figs. 1 and 2.

It can be seen that whether equations (8) or (9) are used for the temperature distribution, the effect of C_0 is most pronounced in the region where $z > z_1$, i.e. in the bulk boiling region. Conversely, in the subcooled region the effect of C_0 is less pronounced.

It was discussed in Section 3.2 that as the process of evaporation takes place, the concentration and flow profiles will change. Consequently, it can be expected that the value of C_0 will change starting with $C_0 \approx 0$ at the point where $\langle\alpha\rangle = 0$, increasing to $C_0 > 1$ in the bulk boiling region. If, in absence of experimental data on flow profiles in the subcooled region, a constant value of C_0 is to be taken for the entire test section, then the results shown in Fig. 6 suggest that this value should correspond to the bulk boiling region.

4.2 *The axial temperature distribution*

Figure 7 shows the computed values $\langle\alpha\rangle$ for given z_0 and C_0 using equation (8) and (9) for the axial temperature distribution. The void fraction computed by assuming thermodynamic equilibrium is also indicated.

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† This may not be true for liquid metals.

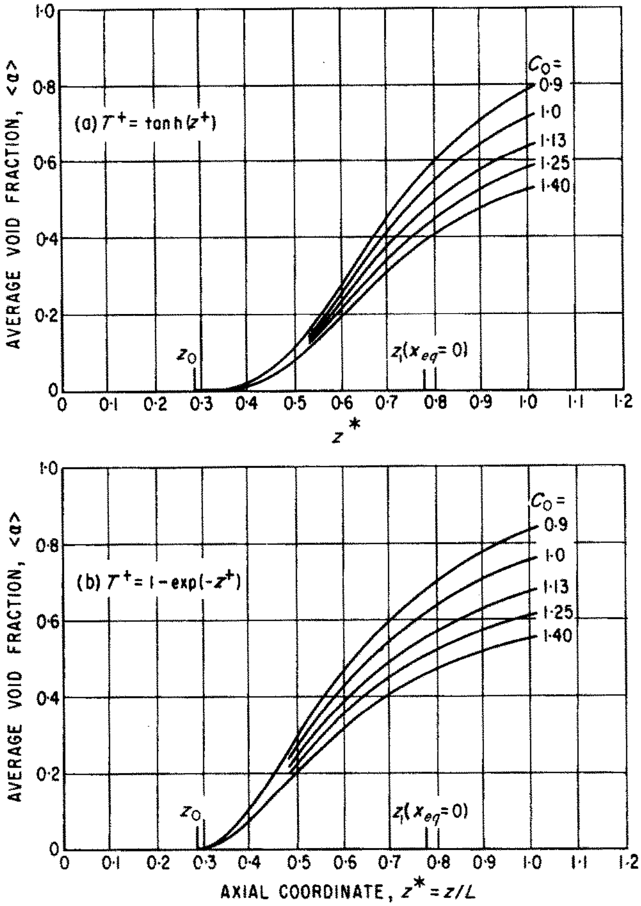

FIG. 6. Effect of distribution parameter C_0 on void formation in subcooled boiling. (a) Using hyperbolic tangent function $T^+ = \tan h (z^+)$; (b) Using exponential function $T^+ = 1 - \exp(-z^+)$.

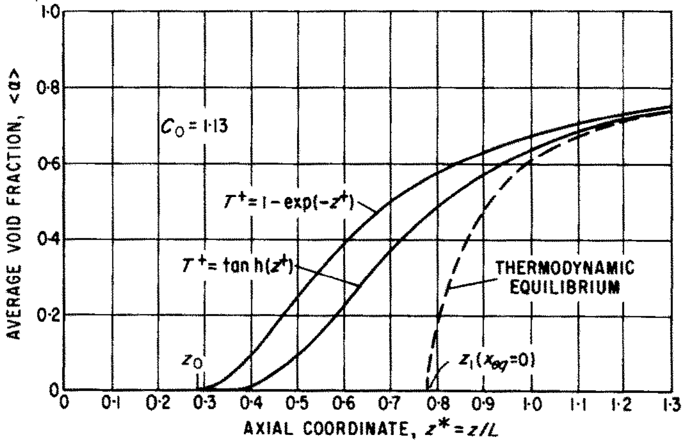

FIG. 7. Effect of axial temperature distribution on computed void fractions in subcooled boiling.

It can be seen from this figure that initially the void fraction computed by using the exponential distribution exhibits a faster increase than that computed by using the hyperbolic tangent distribution. Furthermore, the difference between the two curves could be almost eliminated by an appropriate shift of the point z_0. It is evident that in absence of a reliable method for selecting and predicting z_0 either temperature distribution could be used for predicting $\langle\alpha\rangle$.

It is important to note that in the bulk boiling region the computed $\langle\alpha\rangle$ based on either temperature distribution approaches the value of $\langle\alpha\rangle$ computed by assuming thermodynamic equilibrium. This indicates that the degree of subcooling present has a negligible effect on the void fraction in the bulk boiling region.

4.3 *Point of initial vapor formation*

Figure 8 shows the computed values of $\langle\alpha\rangle$ for a constant C_0 and T^+ as function of the initial point z_0. The void fraction computed assuming thermodynamic equilibrium is also plotted on this figure.

It can be seen from Fig. 8 that, in the bulk boiling region, the effect of accurately locating the point z_0 of initial vapor formation is small and of secondary importance in determining $\langle\alpha\rangle$.

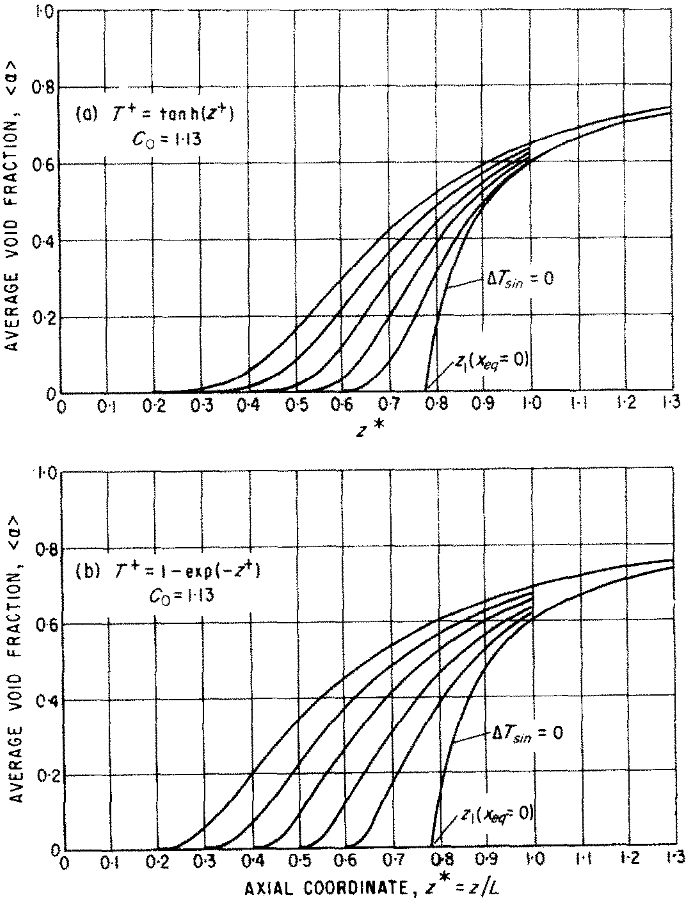

FIG. 8. Effect of point of incipient vapor formation z_0 on void fractions in subcooled boiling. (a) Using hyperbolic tangent function $T^+ = \tan h\,(z^+)$; (b) using exponential function $T^+ = 1 - \exp(-x^+)$.

Apparently, the liquid in the bulk boiling region has a poor memory, i.e. it does not remember the initial state of thermodynamic non-equilibrium.

In the subcooled region however, the effect of z_0 is most pronounced. Indeed, of the three parameters C_0, T^+ and z_0 the last one is definitely the most important. It is evident that the ability to accurately predict the value of $\langle\alpha\rangle$ in the subcooled region depends on the ability to predict z_0 as function of system geometry, fluid properties and operating conditions. It was noted in Section 3.4 that such a criterion is not available at the present time.

It is of interest to note here that in several analyses concerned with the stability of a two phase mixture, it was assumed that the change of z_0 results in a lateral shift of the void curve along the z axis. The results shown in Fig. 8 indicate that such a lateral shift without the attendant change of curvature (as it was assumed in these analyses) is in error.

In the section that follows we shall compare the results predicted by equation (12) to experimental data. In view of the results obtained in this section and since no information is available on the flow profiles in subcooled boiling, the computations will be based on a constant value of C_0 evaluated in the bubbly regime. Furthermore, the values of z_0 will be determined from experimental data. The comparison will indicate therefore with what accuracy one could predict $\langle\alpha\rangle$, assuming C_0 to be constant in equation (12), were a reliable method for predicting z_0 available.

5. COMPARISON WITH EXPERIMENT

Based on the analysis covered in the preceding sections, recent experimental data of several investigators have been evaluated and compared with analytical results. The test sections considered include rectangular, annular, and circular configurations. The test fluids cover water at reduced pressures from 0·005 to 0·22 and Refrigerant-22 at reduced pressures between 0·22 and 0·65.

It was shown in Section 4.1 that the ability to predict accurately the average void fraction $\langle\alpha\rangle$ is most sensitive to the value of the distribution parameter C_0 in the bulk boiling region where $x_{eq} > 0$. In absence of experimental data on flow profiles in subcooled boiling it is desirable therefore to select for C_0 values appropriate to the bubbling flow regime. It is shown in [5] that for this regime the value of C_0 is approximately 1·25 for circular ducts with diameters up to 2·0 in, whereas for rectangular ducts it is equal to 1·5. Actually, somewhat lower values should be expected at moderate and high subcoolings because as profiles vary along the duct, C_0 changes from a value below unity to a value above one. It was recommended in [1, 2, 5] that, in absence of accurate experimental data, values of $C_0 = 1{\cdot}13$ or 1·4 for circular or rectangular geometries respectively be used in equation (1) for predicting $\langle\alpha\rangle$. This value will be used also in the present work in order to test the generality of the correlation.

As outlined in Section 4.1 the axial temperature distribution is not very critical in predicting void fractions. Some of the data have been correlated well with an exponential temperature function; for others, the hyperbolic tangent function appeared to yield better agreement. Unless otherwise noted, the hyperbolic tangent function has generally been used here in comparing analytical predictions with experimental data.

Figure 9 represents a comparison between analysis and experiments for Refrigerant-22 in a circular tube at several reduced pressures. Using a constant distribution parameter $C_0 = 1{\cdot}13$, it was found that at low reduced pressures (Fig. 9a) the predicted void fractions are somewhat high, while at high pressures, the analysis underpredicts the void fractions slightly. This indicates that the distribution parameter C_0 changes with increasing pressure, in agreement with the results, of [2, 3, 5, 6] where it is shown that C_0 tends to unity as the pressure increases. In Fig. 10, the data of Fig. 9(c) are replotted, and compared to computed profiles using a

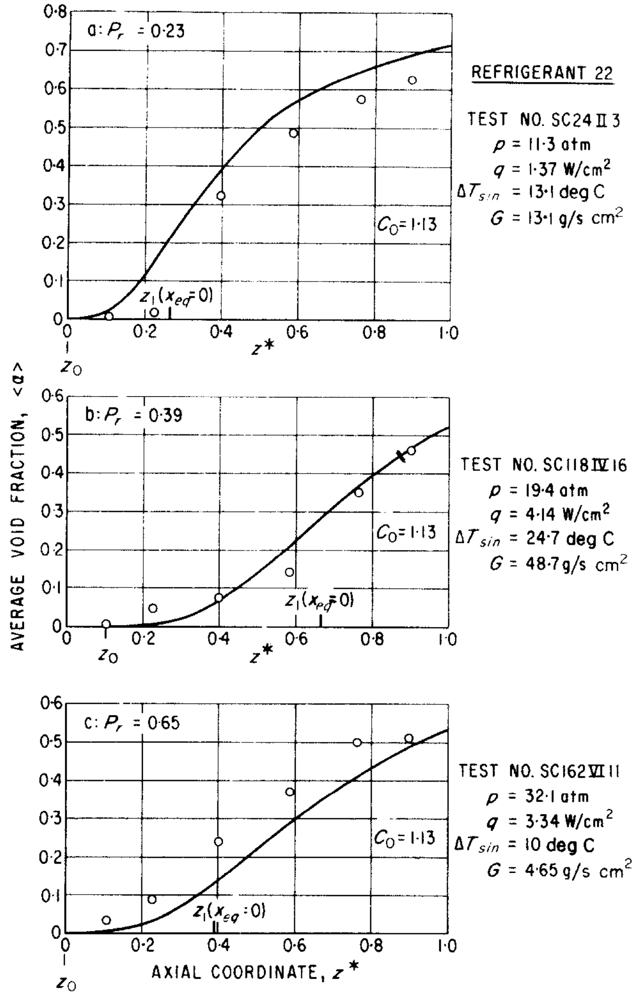

FIG. 9. Comparison of predicted with measured void fractions in subcooled boiling. Authors' data, circular tube 1·01 cm I.D., heated section 154 cm long, Refrigerant-22.

distribution parameter of $C_0 = 1·0$. The results show that either an exponential temperature function with $z_0 = 0$ or a hyperbolic tangent function with a negative value for z_0 can be used to obtain good agreement between test and predictions. A negative value for z_0 would physically imply that voids existed at the inlet to the heated section which is improbable but not impossible with Refrigerant-22 at high pressures and low subcoolings.

Figure 11 shows test results obtained by Firstenberg and Neal [10] in a rectangular channel with water at atmospheric pressure for different power inputs. The values predicted by equation (1) with $C_0 = 1·13$ and hyperbolic tangent temperature distribution are also shown.

As noted above, for rectangular ducts and mixtures with high exit qualities, i.e. with convex concentration profiles the recommended value for C_0 was equal to $C_0 = 1·4$. However, for the tests of [10] with low quality it is probable that

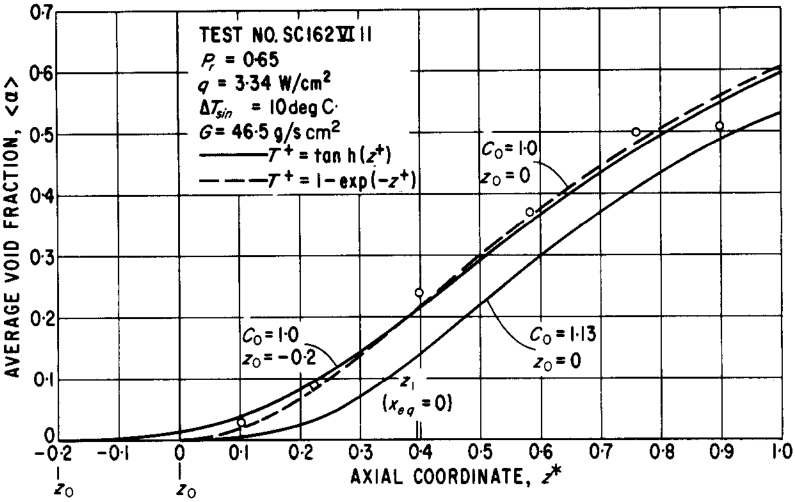

FIG. 10. Comparison of predicted with measured void fractions in subcooled boiling. Authors' data, circular tube 1·01 cm I.D., heated section 154 cm long, Refrigerant-22.

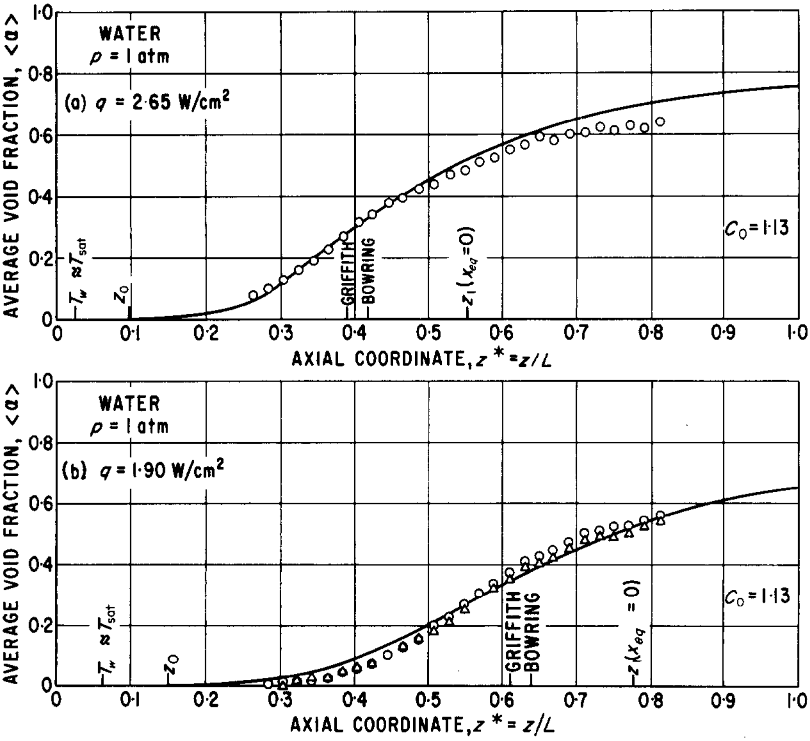

FIG. 11. Comparison of predicted with measured void fractions in subcooled boiling. Data of Firstenberg and Neal [10], Channel 2·11 × 0·295 cm, heated section 62·5 cm long, water, p = 1 atm, $\Delta T_{s\,in}$ = 3·3°C, G = 45·1 g/cm²s.

the profiles were not fully developed resulting in a lower value for C_0. Note also that in Figure 11(a) the void fractions towards the end of the test section may approach the region of fully developed profiles suggesting a value of C_0 larger than 1·13 used in computations, as borne out by experimental data.

It appears from Fig. 11 that the predicted results are in satisfactory agreement with experimental data if the point z_0 of incipient vapor formation is selected correctly. As lower limit, this point has to be downstream of the point where the wall temperature reaches the saturation temperature. On the other hand, nucleation will certainly occur upstream of z_1, where the bulk of the liquid would reach saturation temperature if thermodynamic equilibrium persisted. The exact location where the wall temperature reaches the saturation temperature will depend on the flow and temperature profiles at the inlet to the heated section. For the experiments of [10] the uncertainties in the velocity profile (following a bend and a flow meter) permit only an approximate determination of that point. Inlet effects on the heat-transfer coefficient in this region were computed in accordance with recent results given by Stone [11]. Two experimental criteria given by Griffith [8] and Bowring [9] to identify the point of incipient boiling have been evaluated for the prevailing test conditions, and are indicated in Fig. 11; the agreement with the experimental evidence is not too good. However, both criteria had been obtained in correlating test data at significantly higher pressures, and the disagreement should therefore not be surprising.

It can be seen on Fig. 11 that with the hyperbolic tangent temperature distribution good agreement between predicted and experimental results is obtained if z_0 is located upstream from the point where voids were first detected. Whether this result stems from the assumed temperature profile or whether it implies that voids too small to be detected existed upstream cannot be concluded at this time. Figure 12 shows however, that the results predicted by equation (1) using an exponential temperature distribution (instead of hyperbolic tangent) are even in better agreement with the data of Fig. 11(b) if z_0 is taken in the region where voids are first detected. This result clearly indicates the importance of determining an appropriate and reliable criterion for predicting the point of incipient vapor formation.

Figure 13 shows a comparison between the void fractions predicted by equation (1) and the experimental data of Cook [12] for water at 42 atm., flowing through a rectangular channel.

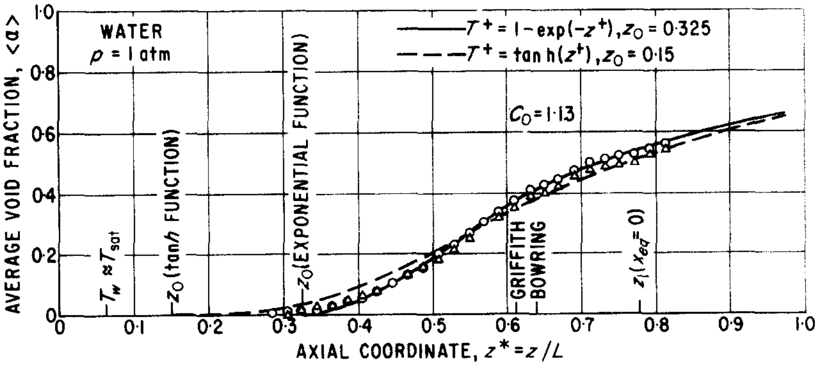

FIG. 12. Comparison of predicted and measured void fractions in subcooled boiling, using different functions for T^+. Data of Firstenberg and Neal [10], channel 2·11 × 0·295 cm, heated section 62·5 cm long, water, $p = 1$ atm, $\Delta T_{s\,in} = 3{\cdot}3°C$, $G = 45{\cdot}1$ g/cm²s, $q = 1{\cdot}9$ Wcm².

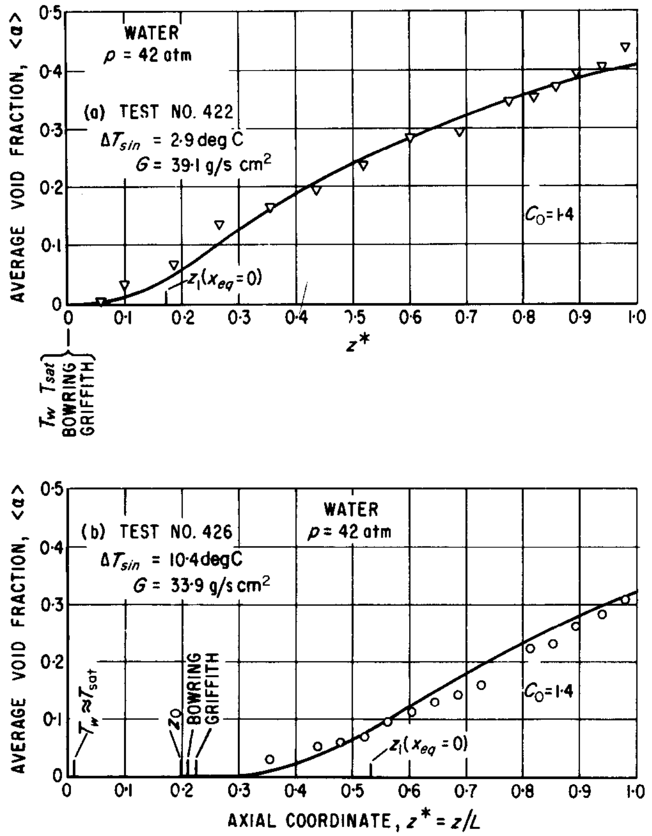

FIG. 13. Comparison of predicted with measured void fractions in subcooled boiling. Data of Cook [12], channel 9·37 × 1·11 cm, heated section 122 cm long, water, $p = 42$ atm, $q = 14{\cdot}7$ W/cm^2.

For both tests, a distribution parameter of 1·4 was used since for each case the major part of the test section was in the region where $x_{eq} > 0$ indicating fully developed convex profile. In Fig. 13(a) with relatively low subcooling, the wall temperature will reach the saturation point at the inlet of the heated section. Both criteria, Bowring's, as well as Griffith's, predict incipient nucleation at the inlet to the test section. With increased subcooling, the point of incipient nucleation is moved downstream as shown for the test reproduced in Fig. 13(b). Here again, both Bowring's and Griffith's criteria predict the point of beginning nucleation very well.

Figure 14 reproduces test data obtained by Rouhani [13] in an annular channel, where the inside perimeter is heated uniformly. The tests were conducted with water 49 atm. Using a distribution parameter of $C_0 = 1{\cdot}13$ in equation (1), good agreement between test and analysis is obtained. The initial point predicted by Bowring's criterion is, however, only in fair agreement with the data. Again, this can be expected, since an annular cross-section with only the inside perimeter being heated falls outside of the geometries covered in deriving the criterion.

It appears from the preceding results that the analytical predictions are in satisfactory agreement with experimental data if the proper value

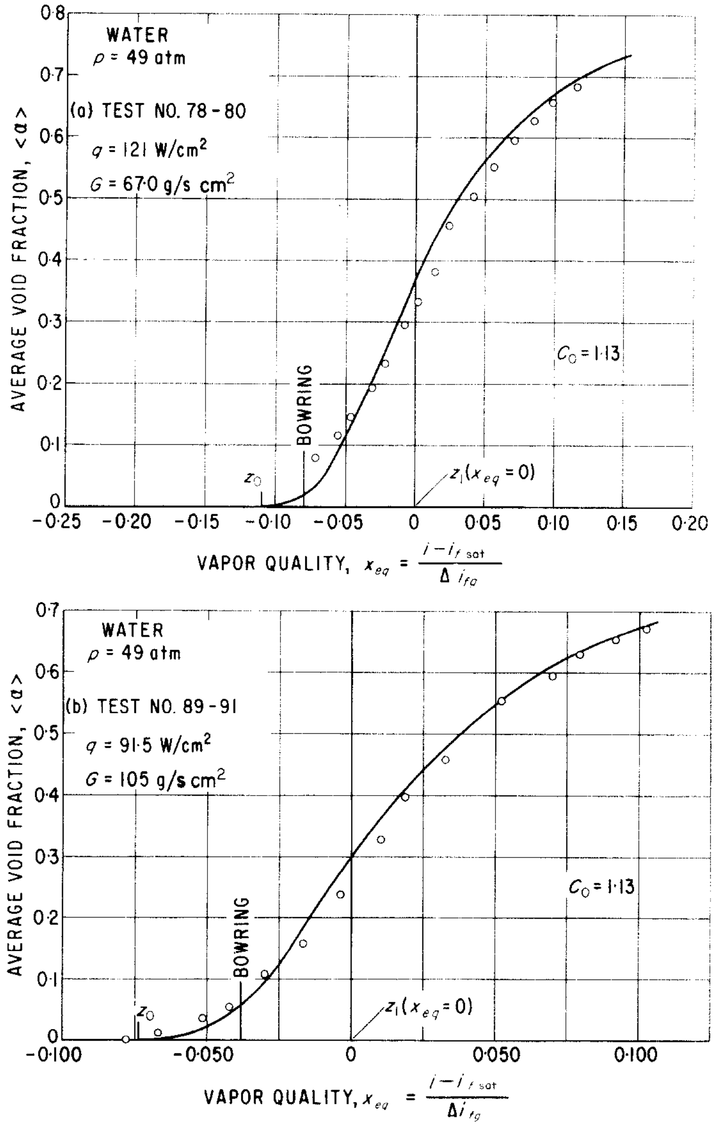

FIG. 14. Comparison of predicted with measured void fractions in subcooled boiling. Data of Rouhani [13], annulus, 1·2 cm I.D., 2·5 cm O.D., water, p = 49 atm.

of z_0 is selected in the subcooled region and C_0 is approximated by the value appropriate to the bulk boiling region. Therefore, in order to predict accurately the void fraction in subcooled boiling it is of utmost importance to derive a reliable criterion for predicting the point z_0 of initial vapor generation. Conversely, for the bulk boiling region it is of primary importance to determine the distribution parameter from measured flow and concentration profiles.

6. THE FORMAL SOLUTION

It was noted in Section 3.3 that in order to compute the void fraction in subcooled boiling it is necessary to determine the vapor mass flow rate G_g given by equation (11). It can be seen that in order to evaluate G_g, it is necessary to determine first the axial temperature distribution T^+. In [1, 2], the latter was assumed to be of the form given by equations (8) and (9); which were obtained by considering the physical boundary conditions that the temperature distribution must satisfy. The formal approach for determining this distribution was also presented in [1, 2]; in what follows we shall discuss some additional aspects and difficulties inherent to this formal method.

Consider a steady, one dimensional flow of a two phase mixture with constant properties and assume that the effects of the kinetic and potential energies can be neglected. Then, the problem of determining the temperature distribution in the liquid can be formulated [1, 2] by considering:

(1) the conservation of mass for the mixture

$$G = G_f + G_g;$$

(2) the continuity equation for the vapor

$$\frac{\partial G_g}{\partial z} = \Gamma_g;$$

(3) the energy equation for the mixture

$$G_f \frac{\partial i_f}{\partial z} + G_g \frac{\partial i_g}{\partial z} + \Gamma_g \Delta i_{fg} = \frac{q\xi_h}{A_c};$$

(4) the equation of state for the vapor

$$i_g = i_g(P, T),$$

and

(5) the constitutive equation of evaporation in the bubbly regime

$$\Gamma_g = \frac{\xi_h}{A_c} \int_{z_0}^{z} \frac{\mathrm{d}m(z, z')}{\mathrm{d}z} J(z')\,\mathrm{d}z'.$$

In the above set we need to discuss only the the last equation because all other relations are self-explaining. The vapor source term Γ_g, represents the rate of vapor mass formation per unit volume. In the bubbly flow regime, it depends on the rate of nucleation $J(z')$ and on the bubble growth $m(z', z)$. The first term represents the rate of bubble nucleation per unit area at a location z' in the duct. The latter describes the mass at a point z in the duct of a bubble that nucleated at z'. Both, the nucleation and the growth depend on the local liquid temperature. Consequently, when both $m(z', z)$ and $J(z')$ are specified, the constitutive equation of evaporation becomes a known function of liquid temperature which, in turn, varies with distance.

It can be seen that the problem is defined by five equations in terms of five variables, i.e. G_f, G_g, Γ_g, i_f and i_g. Since all of these variables are functions of temperature, the substitution of four of these equations into the energy equation yields an integro-differential equation in terms of the liquid temperature. If we assume that the vapor in the subcooled boiling region remains at saturation temperature, then this equation takes the form of:

$$\left[G - \frac{\xi_h}{A_c} \int_{z_0}^{z} m(z', z)\, J(z')\,\mathrm{d}z'\right] c_p \frac{\mathrm{d}T_L}{\mathrm{d}z} + \left[\frac{\xi_h}{A_c} \int_{z_0}^{z} \frac{\mathrm{d}m}{\mathrm{d}z}(z', z)\, J(z')\,\mathrm{d}z'\right] \Delta i_{fg} = \frac{q\xi_h}{A_c}.$$

Thus, when the bubble nucleation and growth are known functions of temperature, the solution of this equation will yield the axial temperature distribution in the liquid. By substituting this distribution in equation (11) one obtains the desired vapor mass flux appropriate to the particular constitutive equation of evaporation for the bubbly flow regime.

The difficulty in following this formal approach at this time stems from the uncertainties related to: (1) the rate of heterogeneous nucleation $J(z')$; (2) the nucleating characteristics of the surface which affect $J(z')$, and (3) the bubble growth law $m(z', z)$. It was noted in

[1, 2] that at the present time none of these parameters is known with sufficient accuracy to warrant the use of the formal approach to obtain the temperature distribution. Although several assumptions could be made to first evaluate the constitutive equation of evaporation and then to integrate the integro-differential equation for the temperature distribution, experimental data are not available which could be used to support these assumptions. Instead of using several assumptions pertaining to $J(z')$ and $m(z', z)$, only one assumption was introduced in [1, 2], i.e. the form of the temperature function was assumed directly. In addition to simplicity, this latter approach has two advantages at present. First, the boundary conditions for the temperature distribution are known and two, it is always more desirable to reduce the number of assumptions that are required to make a problem tractable.

The results presented in this paper indicate that the accurate determination of the initial point z_0 is by far more important than the precise specification of the temperature distribution in subcooled boiling. Consequently, the exponential or the hyperbolic tangent approximation for the temperature distribution may prove to be sufficiently accurate to reduce the incentive for determining the constitutive equation of evaporation. However, if investigations in the future reveal that a change of surface conditions or the introduction of additives changes the void fraction in the subcooled region, then the determination of the appropriate constitutive equation will be required. This statement follows from the fact that the rate of heterogeneous nucleation $J(z')$ is affected by the surface conditions of the heated wall as well as by the additives. It can be seen from the integro-differential equation that these will influence therefore the temperature distribution in the subcooled region. Consequently, different distributions T^+ could be expected for different nucleating characteristics.

It appears therefore that variations of surface conditions and introduction of additives may have two effects on the average void fraction. One may result from the shift of the point of initial vapor formation z_0 in equation (1); the other from a change of the temperature distribution T^+. Whether this is indeed the case remains to be shown by future experiments.

7. CONCLUSIONS

(1) It was shown previously [1, 2] that four effects must be taken into account in an analysis concerned with predicting the vapor void fraction in subcooled boiling. These are the effects of: (1a) the local relative velocity; (2b) the concentration profiles; (3c) the flow profile, and (4d) the axial temperature profile. In [1, 2] these four effects were accounted for by means of the vapor drift velocity V_{gj}, the distribution parameter C_0, the dimensionless axial temperature distribution T^+ and the point of initial vapor formation z_0.

(2) In this paper the effects of these four parameters on the ability to predict accurately the void fraction in subcooled boiling has been evaluated and the results have been compared to recent experimental data.

(3) It is shown that the effect of the distribution parameter C_0 is most pronounced in the bulk boiling region. Consequently, if in absence of experimental data on flow profiles in the subcooled region, a constant value of C_0 is to be taken for the entire test section then results of this paper indicate that this value of C_0 should correspond to the bulk boiling region.

(4) The vapor drift velocity has only a minor effect in forced convection systems with high mass flow rates.

(5) The point of incipient vapor formation z_0 greatly affects the predicted void fraction in the subcooled regime. No reliable criterion for predicting the location of this point exists at this time. It is concluded therefore, that further work to define and predict its location is mandatory for reliable and accurate prediction of void fractions in subcooled boiling.

(6) The axial temperature distribution has been approximated by different functions that

satisfy the physical boundary conditions. It appears that the precise specification of the temperature distribution is of secondary importance when compared to the need of locating properly the point of initial vapor formation. The profiles used up to date give satisfactory results when compared to available experimental data. However, final conclusion on this point has to be reserved until a method for predicting z_0 becomes available.

(7) It appears that the previous history of the liquid in the subcooled region has only a minor effect on the void fraction in the bulk region.

(8) The results of the analysis show that changing the conditions of the heated surface and the introduction of additives may have two effects on the void fraction in subcooled boiling. These effects are discussed and a method for taking them into account is presented.

(9) Satisfactory agreement is shown between predicted void fractions and recent experimental data obtained in subcooled boiling of Refrigerant-22 and of water over a pressure range given by $0{\cdot}22 \leqslant p_r \leqslant 0{\cdot}65$ and $0{\cdot}005 \leqslant p_r \leqslant 0{\cdot}22$ respectively.

(10) The results presented in this paper indicate that for the bulk boiling region, future investigations should be directed toward determining the flow and concentration profiles which specify the distribution parameter C_0. In the subcooled boiling region however, the primary emphasis should be placed on determining the point of incipient vapor formation as function of fluid properties, system geometry and operating characteristics.

ACKNOWLEDGEMENT

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NOTE ADDED TO PROOF

We observed in Section 3.3 and G. E. Dix† has informed us that the analysis of [1, 2] could have been expressed also in terms of enthalpy or of quality distribution instead of a temperature distribution given by equation (8) or equation (9).

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† Atomic Power Equipment Department, General Electric Company, San Jose, Calif.

For example, equation (8) when expressed in terms of enthalpy becomes

$$i_f^+ = \frac{i_f(z) - i_f(z_0)}{i_f(z_1) - i_f(z_0)}$$

$$= 1 - \exp\left\{-\frac{i(z) - i_f(z_0)}{i_f(z_1) - i_f(z_0)}\right\}$$

$$= 1 - \exp\left\{-\frac{\int_{z_0}^{z} q(z)\,\mathrm{d}z}{\int_{z_0}^{z_1} q(z)\,\mathrm{d}z}\right\} \qquad (16)$$

where $i_f(z_1)$ and $i_f(z_0)$ are the liquid enthalpies at saturation and at the point of initial vapor formation respectively.

Similarly, using the standard definition of the quality:

$$x = \frac{i(z) - i_f(z)}{i_g(z) - i_f(z)} \qquad (17)$$

and of equilibrium quality:

$$x_{eq} = \frac{i(z) - i_f(z_1)}{\Delta i_{fg}} \qquad (18)$$

equation (8), i.e. equation (16) can be expressed in terms of quality distribution, thus

$$1 + \left[\frac{\Delta i_{fg}}{c_p\,\Delta T_s}\right]\frac{x - x_{eq}}{1 - x}$$

$$= 1 - \exp\left\{-\frac{x_{eq}\,\Delta i_{fg}}{c_p\,\Delta T_s} - 1\right\} \qquad (19)$$

whence

$$x = \frac{x_{eq} + \dfrac{c_p\,\Delta T_s}{\Delta i_{fg}}\exp\left\{-\dfrac{x_{eq}\,\Delta i_{fg}}{c_p\,\Delta T_s} - 1\right\}}{1 + \dfrac{c_p\,\Delta T_s}{\Delta i_{fg}}\exp\left\{-\dfrac{x_{eq}\,\Delta i_{fg}}{c_p\,\Delta T_s} - 1\right\}} \qquad (20)$$

which is of the form of equation (13).

Following the analysis of [1, 2] and using the method developed there, Levy [14] reported recently an analysis concerned with predicting the void fraction in sub-cooled boiling. Levy derives an equation for determining the point of vapor formation z_0. However, in evaluating the effects of thermodynamic non-equilibrium he replaces equation (8) with an assumed quality distribution given by

$$x = x_{eq} + \frac{c_p\,\Delta T_s}{\Delta i_{fg}}\exp\left\{-\frac{x_{eq}\,\Delta i_{fg}}{c_p\,\Delta T_s} - 1\right\}. \qquad (21)$$

Although it was not noted by Levy, it can be seen by comparing equation (21) with equation (20), that equation (21) is only an approximation. Furthermore, equation (21) *cannot* be derived from the standard definition of quality.

The following two conclusions were made by Levy:

(1) "A method was developed to predict vapor volumetric fractions during subcooled forced convection boiling.

(2) "The method gives general agreement with available data."

We subscribe to the first conclusion, a method is available for predicting the void fraction in subcooled boiling: the formulation and results of [1, 2] can be used to account for the effects of: (a) non-uniform void and flow distributions (by means of C_0); (b) relative velocity (by means of V_{gj}) and (c) thermal non-equilibrium (by means of an axial temperature or quality distribution) whereas the relation of Levy can be used to predict the point of initial vapor formation z_0.

We subscribe also to the second conclusion and it would seem to us that the effort originally planned for the solution of the problem concerned with the steady-state void fraction in subcooled boiling, could be redirected now toward more critical and significant problem areas.

Résume—On présente une évaluation des effets de différents paramètres sur la possibilité de prévoir de façon précise la concentration volumique de vapeur dans l'ébullition sous-refroidie. On montre que (1) le paramètre de distribution est d'importance primordiale lorsqu'on s'approche de l'équilibre thermodynamique; (2) le point du début de formation de la vapeur affecte beaucoup la fraction de vides dans le régime sous refroidi—aucune méthode sûre de prévision de ce point n'existe actuellement; et (3) il apparait que la distribution axiale de température a un effet mineur.

On conclut donc, que la possibilité de prévoir le point du début de formation de la vapeur est impérative pour une prévision précise de la fraction de vides dans l'ébullition sous-refroidie. La recherche future devrait ainsi être dirigée vers la définition et la prévision de l'endroit de ce point.

Les résultats de la théorie montrent également que les conditions de la surface chauffée et l'introduction d'additifs peuvent avoir deux effets sur la fraction de vides dans l'ébullition sous-refroidie. Ces effets sont discutés et l'on présente une méthode pour en tenir compte.

On montre qu'il y a un accord satisfaisant entre les fractions de vides prédites et les récents résultats expérimentaux obtenus dans l'ébullition sous-refroidie de Réfrigérant-22 et d'eau dans une large gamme de pressions.

Zusammenfassung—Eine Abschätzung des Einflusses verschiedener Parameter über die Möglichkeit, die volumetrische Dampfkondensation beim Sieden von unterkühlter Flüssigkeit vorauszusagen, wird hier angegeben. Es wird gezeigt, dass: (1) der Verteilungsparameter umso grössere Bedeutung gewinnt, je mehr das thermodynamische Gleichgewicht erreicht wird; (2) die Stelle erster Dampfbildung den Dampfanteil beim unterkühlten Regime stark beeinflusst,—zur Voraussage dieser Stelle gibt es gegenwärtig keine zuverlässige Methode—(3) die Achsialtemperaturverteilung von geringerer Bedeutung zu sein scheint.

Es wird daraus geschlossen, dass zur genauen Errechnung des Dampfanteiles beim Sieden unterkühlter Flüssigkeit die Bestimmung der Stelle erster Dampfbildung erforderlich ist. Zukünftige Untersuchungen sollen darauf gerichtet sein, die Lage dieser Stelle zu definieren und zu ermitteln.

Die Ergebnisse der Analyse zeigen auch, dass die Bedingungen der beheizten Oberfläche und die Einführung additiver Glieder zwei Einflüsse auf den Dampfanteil beim Sieden unterkühlter Flüssigkeit bewirken können. Diese Einflüsse werden diskutiert und eine Methode zu ihrer Berücksichtigung wird angegeben.

Zufriedenstellende Übereinstimmung ergibt sich zwischen den vorherbestimmten Dampfanteilen und kürzlich gewonnenen Versuchswerten beim Sieden von unterkühltem R 22 und von Wasser in einem weiten Druckbereich.

Аннотация—Дана оценка влияния различных параметров на возможность точного расчета объемной концентрации пара при кипении недогретой жидкости. Показано, что (1) по мере приближения к состоянию термодинамического равновесия становится доминирующим параметр распределения; (2) в режиме кипения недогретой жидкости точка начального парообразования оказывает значительное влияние на относительное объемное паросодержание — в настоящее время ещё нет надежного метода расчета данной точки; и (3) оказывается, что аксиальное распределение температуры имеет меньшее значение.

Сделан вывод, что для расчета объемного паросодержания, при кипении недогретой жидкости необходимо уметь рассчитывать точку начального парообразования. Поэтому дальнейшие исследования должны быть направлены на определение и расчет местоположения данной точки.

Результаты анализа также показывают, что при кипении недогретой жидкости состояние нагретой поверхности и добавление присадок могут оказывать двоякое влияние на относительное объемное паросодержание. Эти эффекты рассмотрены и дан метод их учета.

Показано удовлетворительное соответствие между расчетными значениями относительного объемного паросодержания и последними экспериментальными данными, полученными при кипении холодильного агента 22 и воды в широком диапазоне давлений.